**Topic: Assignment Problems**

Time: 45 mins Marks: /45 marks

**Calculator Assumed**



**Question One: [3, 4: 7 marks]**

a) The following table show the number of vehicles which will be effected by road works on road A, B and C depending on the rerouting of traffic via route 1, 2 or 3.

Assign roads A, B and C to the rerouting route in order to minimise the number of vehicles effected.

State which road should use which route and the total number of vehicles expected to be effected.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Route 1** | **Route 2** | **Route 3** |
| **Road A** | 110 | 200 | 260 |
| **Road B** | 100 | 300 | 210 |
| **Road C** | 190 | 250 | 220 |

b) A local government has projects A, B, C and D which need to be completed. They have quotes from four contractors and need to assign each contractor one project.

Which project should be assigned to each contractor in order to minimise costs to the government and what is the total cost of project A, B, C and D.

Costs for each project by each contractor are shown in the table below in $1 000’s.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Contractor 1** | **Contractor 2** | **Contractor 3** | **Contractor 4** |
| **Project A** | 360 | 210 | 100 | 190 |
| **Project B** | 250 | 220 | 200 | 90 |
| **Project C** | 250 | 200 | 240 | 210 |
| **Project D** | 205 | 250 | 300 | 200 |

**Question Two: [1, 3, 8: 12 marks]**

Four people needing a kidney transplant and four willing donors are ranked according to the potential for a successful transplant. 0 being not a successful match and 100 being a perfect match. The following table shows a summary of the results.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Donor 1** | **Donor 2** | **Donor 3** |
| **Recipient A** | 26 | 29 | 30 |
| **Recipient B** | 58 | 55 | 56 |
| **Recipient C** | 75 | 78 | 69 |

a) If a computer randomly allocated recipients to donors and the allocation is shown below, calculate the potential success rate out of 300.



b) Is there a better allocation of recipients to donors in order to maximise the chance of success?

c) If another recipient and another donor are included in the process show how the Hungarian algorithm can be used to identify the maximum chances of success by matching the recipients to the donors.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Donor 1** | **Donor 2** | **Donor 3** | **Donor 4** |
| **Recipient A** | 26 | 29 | 30 | 25 |
| **Recipient B** | 58 | 55 | 56 | 60 |
| **Recipient C** | 75 | 70 | 69 | 70 |
| **Recipient D** | 66 | 68 | 68 | 65 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Donor 1** | **Donor 2** | **Donor 3** | **Donor 4** |
| **Recipient A** |  |  |  |  |
| **Recipient B** |  |  |  |  |
| **Recipient C** |  |  |  |  |
| **Recipient D** |  |  |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Donor 1** | **Donor 2** | **Donor 3** | **Donor 4** |
| **Recipient A** |  |  |  |  |
| **Recipient B** |  |  |  |  |
| **Recipient C** |  |  |  |  |
| **Recipient D** |  |  |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Donor 1** | **Donor 2** | **Donor 3** | **Donor 4** |
| **Recipient A** |  |  |  |  |
| **Recipient B** |  |  |  |  |
| **Recipient C** |  |  |  |  |
| **Recipient D** |  |  |  |  |

**Question Three: [4, 3: 7 marks]**

a) Find the perfect matching of minimum weighting for the following complete bipartite graph.



b) Find the perfect matching of maximum weighting for the complete bipartite graph above.

**Question Four: [1, 3, 3: 7 marks]**

The following table displays the delivery costs for companies A, B, C and D to complete four deliveries.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** |
| **A** | 100 | 220 | 350 | 110 |
| **B** | 90 | 200 | 300 | 100 |
| **C** | 110 | 190 | 360 | 150 |
| **D** | 150 | 140 | 310 | 120 |

The Hungarian algorithm was being used to find the allocation which would minimise delivery costs.

The first step of the Hungarian algorithm is shown in the partially completed table below.

a) Complete this step by completing the table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** |
| **A** | 0 | 120 | 250 | 10 |
| **B** | 0 | 110 | 210 | 10 |
| **C** | 0 | 80 | 250 | 40 |
| **D** |  |  |  |  |

Further along in the process of the Hungarian algorithm the table is as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** |
| **A** | 0 | 100 | 60 | 10 |
| **B** | 0 | 90 | 20 | 10 |
| **C** | 0 | 60 | 60 | 40 |
| **D** | 30 | 0 | 0 | 0 |

b) Complete the final stages of the Hungarian algorithm to determine the minimum allocation.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** |
| **A** |  |  |  |  |
| **B** |  |  |  |  |
| **C** |  |  |  |  |
| **D** |  |  |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** |
| **A** |  |  |  |  |
| **B** |  |  |  |  |
| **C** |  |  |  |  |
| **D** |  |  |  |  |

c) Indicate the allocation on the original table and state the total cost of the four deliveries.

**Question Five: [1, 4, 4, 3: 12 marks]**

Consider the following table:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **1** | **2** | **3** |
| **A** |  |  |  |
| **B** |  |  |  |
| **C** |  |  |  |

a) If is an integer , which expression represents the highest value?

b) If we want to maximise the allocation on the table above show the first step of the Hungarian algorithm in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **1** | **2** | **3** |
| **A** |  |  |  |
| **B** |  |  |  |
| **C** |  |  |  |

c) Show that the final table for maximizing this allocation using the Hungarian algorithm is shown in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **1** | **2** | **3** |
| **A** |  |  |  |
| **B** |  |  |  |
| **C** |  |  |  |

The sum of the final allocation which maximise the result is 33.

d) Calculate the value of *x*.

**Topic: SOLUTIONS**

Time: 45 mins Marks: /45 marks

**No calculator allowed**



**Question One: [3, 4: 7 marks]**

a) The following table show the number of vehicles which will be effected by road works on road A, B and C depending on the rerouting of traffic via route 1, 2 or 3.

Assign roads A, B and C to the rerouting route in order to minimise the number of vehicles effected.

State which road should use which route and the total number of vehicles expected to be effected.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Route 1** | **Route 2** | **Route 3** |
| **Road A** | 110 | 200 | 260 |
| **Road B** | 100 | 300 | 210 |
| **Road C** | 190 | 250 | 220 |



Road A – Route 2

Road B – Route 1 Total vehicles effected 520

Road C – Route 3

b) A local government has projects A, B, C and D which need to be completed. They have quotes from four contractors and need to assign each contractor one project.

Which project should be assigned to each contractor in order to minimise costs to the government and what is the total cost of project A, B, C and D.

Costs for each project by each contractor are shown in the table below in $1 000’s.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Contractor 1** | **Contractor 2** | **Contractor 3** | **Contractor 4** |
| **Project A** | 360 | 210 | 100 | 190 |
| **Project B** | 250 | 220 | 200 | 90 |
| **Project C** | 250 | 200 | 240 | 210 |
| **Project D** | 205 | 250 | 300 | 200 |

PA – C3, PB – C4, PC – C2, PD – C1 

Total cost

**Question Two: [1, 3, 8: 12 marks]**

Four people needing a kidney transplant and four willing donors are ranked according to the potential for a successful transplant. 0 being not a successful match and 100 being a perfect match. The following table shows a summary of the results.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Donor 1** | **Donor 2** | **Donor 3** |
| **Recipient A** | 26 | 29 | 30 |
| **Recipient B** | 58 | 55 | 56 |
| **Recipient C** | 75 | 78 | 69 |

a) If a computer randomly allocated recipients to donors and the allocation is shown below, calculate the potential success rate out of 300.

rate

b) Is there a better allocation of recipients to donors in order to maximise the chance of success?



Yes A – 3 🡪 30

 B – 1 🡪 58

 C – 2 🡪 78 rate

c) If another recipient and another donor are included in the process show how the Hungarian algorithm can be used to identify the maximum chances of success by matching the recipients to the donors.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Donor 1** | **Donor 2** | **Donor 3** | **Donor 4** |
| **Recipient A** | 26 | 29 | 30 | 25 |
| **Recipient B** | 58 | 55 | 56 | 60 |
| **Recipient C** | **75** | 70 | 69 | 70 |
| **Recipient D** | 66 | 68 | 68 | 65 |



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Donor 1** | **Donor 2** | **Donor 3** | **Donor 4** |
| **Recipient A** | 49 | 46 | 45 | 50 |
| **Recipient B** | 17 | 20 | 19 | 15 |
| **Recipient C** | 0 | 5 | 6 | 5 |
| **Recipient D** | 9 | 7 | 7 | 10 |



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Donor 1** | **Donor 2** | **Donor 3** | **Donor 4** |
| **Recipient A** | 4 | 1 | 0 | 5 |
| **Recipient B** | 2 | 5 | 4 | 0 |
| **Recipient C** | 0 | 5 | 6 | 5 |
| **Recipient D** | 2 | 0 | 0 | 3 |



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Donor 1** | **Donor 2** | **Donor 3** | **Donor 4** |
| **Recipient A** | 4 | 1 | 0 | 5 |
| **Recipient B** | 2 | 5 | 4 | 0 |
| **Recipient C** | 0 | 5 | 6 | 5 |
| **Recipient D** | 2 | 0 | 0 | 3 |

D1 – C

D2 –D

D3 – A

D4 – B

**Question Three: [4, 3: 7 marks]**

a) Find the perfect matching of minimum weighting for the following complete bipartite graph.



|  |  |  |  |
| --- | --- | --- | --- |
|  | **A** | **B** | **C** |
| **1** | ~~20~~ ~~12~~ 8 | ~~15~~ ~~10~~ 6 | ~~6~~ ~~4~~ 0 |
| **2** | ~~10~~ 2 | ~~11~~ 6 | ~~2~~ 0 |
| **3** | ~~8~~ 0 | ~~5~~ 0 | ~~3~~ 1 |

|  |  |  |  |
| --- | --- | --- | --- |
|  | **A** | **B** | **C** |
| **1** | ~~8~~ 6 | ~~6~~ 4 | 0 |
| **2** | ~~2~~ 0 | ~~6~~ 4 | 0 |
| **3** | 0 | 0 | ~~1~~ 3 |

A – 3 6

B – 1 10 21

C – 2 5

b) Find the perfect matching of maximum weighting for the complete bipartite graph above.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **A** | **B** | **C** |
| **1** | ~~20~~ 0 | ~~15~~ 5 | ~~6~~ 14 |
| **2** | ~~10~~ 10 | ~~11~~ 9 | ~~2~~ 18 |
| **3** | ~~8~~ 12 | ~~5~~ 15 | ~~3~~ 17 |

|  |  |  |  |
| --- | --- | --- | --- |
|  | **A** | **B** | **C** |
| **1** | 0 | 5 | ~~14~~ 9 |
| **2** | ~~10~~ 1 | ~~9~~ 0 | ~~18 9~~ 4 |
| **3** | ~~12~~ 0 | ~~15~~ 3 | ~~17 5~~ 0 |

A – 1 20

B – 2 11 34

C – 3 3

**Question Four: [1, 3, 3: 7 marks]**

The following table displays the delivery costs for companies A, B, C and D to complete four deliveries.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** |
| **A** | 100 | 220 | 350 | 110 |
| **B** | 90 | 200 | 300 | 100 |
| **C** | 110 | 190 | 360 | 150 |
| **D** | 150 | 140 | 310 | 120 |

The Hungarian algorithm was being used to find the allocation which would minimise delivery costs.

The first step of the Hungarian algorithm is shown in the partially completed table below.

a) Complete this step by completing the table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** |
| **A** | 0 | 120 | 250 | 10 |
| **B** | 0 | 110 | 210 | 10 |
| **C** | 0 | 80 | 250 | 40 |
| **D** | 30 | 20 | 190 | 0 |

Further along in the process of the Hungarian algorithm the table is as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** |
| **A** | 0 | 100 | 60 | 10 |
| **B** | 0 | 90 | 20 | 10 |
| **C** | 0 | 60 | 60 | 40 |
| **D** | 30 | 0 | 0 | 0 |

b) Complete the final stages of the Hungarian algorithm to determine the minimum allocation.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** |
| **A** | 0 | ~~90~~ 80 | ~~50~~ 40 | 0 |
| **B** | 0 | ~~80~~ 70 | ~~10~~ 0 | 0 |
| **C** | 0 | ~~50~~ 40 | ~~50~~ 40 | 30 |
| **D** | 40 | 0 | 0 | 10 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** |
| **A** |  |  |  | 0 |
| **B** |  |  | 0 |  |
| **C** | 0 |  |  |  |
| **D** |  | 0 |  |  |

c) Indicate the allocation on the original table and state the total cost of the four deliveries.

A – 4 110

B – 3 300

C – 1 110 Total cost $660

D – 2 140

**Question Five: [1, 4, 4, 3: 12 marks]**

Consider the following table:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **1** | **2** | **3** |
| **A** |  |  |  |
| **B** |  |  |  |
| **C** |  |  |  |

a) If is an integer , which expression represents the highest value?

b) If we want to maximise the allocation on the table above show the first step of the Hungarian algorithm in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **1** | **2** | **3** |
| **A** |  |  |  |
| **B** |  |  |  |
| **C** |  |  |  |

c) Show that the final table for maximizing this allocation using the Hungarian algorithm is shown in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **1** | **2** | **3** |
| **A** |  |  |  |
| **B** |  |  |  |
| **C** |  |  |  |



The sum of the final allocation which maximise the result is 33.

d) Calculate the value of *x*.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **1** | **2** | **3** |
| **A** | 10 | 8 | 6 |
| **B** | 9 | 12 | 10 |
| **C** | 7 | 11 | 11 |